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Solving and Modeling Ken-ken Puzzle byUsingHybridGeneticsAlgorithm

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Abstract— **Kenken is logic *puzzle* which similar to Sudoku. The basic rules are same as Sudoku. As in sudoku, the goal of each puzzle is to fill a grid with digits 1 through 4 for a 4×4 grid, 1 through 5 for a 5×5, etc. No digit appears more than once in any row or column. Grids range in size from 3×3 to 9×9. Unlike sudoku, KenKen grids are divided into heavily outlined groups of cells often called "cages" and the numbers in the cells of each cage must produce a certain "target" number when combined using a specified mathematical operation (addition, subtraction, multiplication or division).). For example, a three-cell cage specifying addition and a target number of 6 in a 4×4 puzzle might be satisfied with the digits 1, 2, and 3. Digits may be repeated within a cage, as long as they are not in the same row or column. No operation is relevant for a single-cell cage: placing the "target" in the cell is the only possibility. The target number and operation appear in the upper left-hand corner of the cage. Initially, none of the cell in a cage in Kenken has value. Kenken is such a simple logic *puzzle*, but finding the solution is quite complex, especially for harder problem. Therefore, research is conducted to develop software which can solve Kenken problem.**

The research will produce software that can solve kenken puzzle byusingheuristic search and hybrid genetic algorithm. Heuristic search will be done by applying certain logic rules. If the puzzle is still not being able to be solved then hybrid genetic algorithm will be run. How can the software solve the kenken problem will be explained in this paper.

After conducting some researchs and implementing the software and doing the testing, some experiments were conducted and it proved that the hybrid genetics algorithm can solve the kenken puzzle and bbased on experiment result, the size of *puzzle* and the difficulty of problems affect the time needed to find a solution

Keywords—**Kenken logic puzzle, heuristics, genetics algorithm**

I. INTRODUCTION

As a kind of arithmetic and grid puzzle, Kenken is also recognized as KenDoku, Calcudoku atau Mathdoku. It was invented in 2004 by the Japanese math teacher Tetsuya Miyamoto. The name derives from the Japanese word for cleverness.

As in sudoku, the goal of each puzzle is to fill a grid with digits *1* through *n* for a $n \times n$ grid, so that no digit appears more than once in any row or column. Grids range in size from 3×3 to 9×9. Additionally, KenKen grids are divided into heavily outlined groups of cells often called "cages" and the numbers in the cells of each cage must produce a certain "target" number when combined using a specified mathematical operation (either addition, subtraction, multiplication or division). If the operation is subtraction or division then the cage has to consist of two cells. The target number and operation appear in the upper left-hand corner of the cage. Figure 1 is an example of Kenken*6x6*.

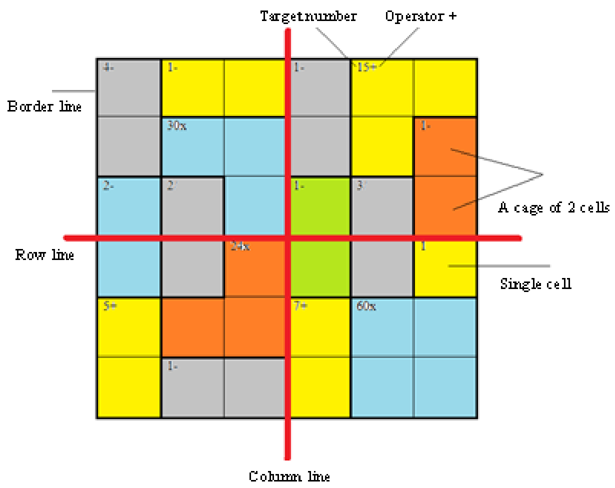

Figure 1. A sampleKenkenpuzzle 6 x 6

There are no many papers discussing on how to solve Kenken puzzle. However, some logical rules can be applied. There are Singlesquare, naked pair, nakedtriple, eviltwin, Hidden single, Killer combination, and X-wing.

Through this paper will be discussed on how to implement the rules so that computer can solve the problems being generated by Will Shortz in Kenken Pro 2 game.

II. HEURISTICS SEARCH AND GENETIC ALGORITHM

Heuristic search is an AI search technique that employs heuristic for its moves. *Heuristic* is a rule of thumb that probably leads to a solution. Heuristics play a major role in search strategies because of exponential nature of the most problems. Heuristics help to reduce the number of alternatives from an exponential number to a polynomial number. In a general sense, the term heuristic is used for any advice that is often effective, but is not guaranteed to work in every case.

There are some heuristics search techniques: Generate and Test, Hill Climbing, and Best First Search. This paper discusses generate and test through genetic algorithm.

A. Rules based system in Kenken

Some logic rules are also used in solving Kenken puzzle. They are Single square, nakedpair, nakedtriple, eviltwin, Hidden single, Killer combination, and X-wing. Single square rule is applied if a cage has only one cell. It means the value of that cell is same as a target number.

Naked pair rule is used if there are two cells in a row or in a column that have precisely the same two possible values to fill the cells. It means other cells in that row or column respectively has no chance to have the same possible value with that of that the two cells have. Figure 2 represents on how this naked pair rule works. The cells marked with green precisely have two possible values (1 or 7). These are a naked pair. Since 1 and 7 must occur in these two cells, we can eliminate 1 and 7 from the cells marked with red.

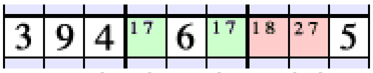

Figure 2. An example on how to detect Naked pair Rule

Naked triple works identical with naked pair but the number of cell is three instead of two. It implies the number of possible values to fill the cells is also three.

Evil twin is the easiest rule. If a cage consistsof two cells and whenever one cell is solved then the other cell is easy - it is simply the value is needed to make the sum work out. For example from Figure 3, once cell in the bottom left corner is solved with the- 4, then the square above it is 9. In fact, the rule can be generalized for larger areas. The last unsolved cell in an area is simply the value needed to make the sum work out.

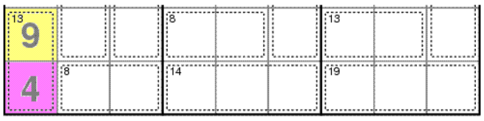

Figure 3.An example of Evil Twin Rule.

Hidden singles are a simple enough concept, but are often quite hard to spot.The definition of a hidden single is when a cell is the only one of its possible value in an entire row, column.From figure 4, the possible values to solve the most left cell are 3, 5 or 7. However this row must have a “7” in it somewhere. It can be seen that only that cell that has a “7” as one of its candidates. That is hidden single. That cell should be solved with “7”.

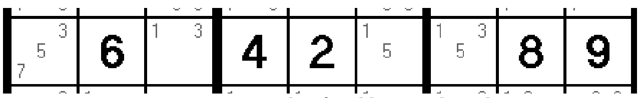

Figure 4. An example of Hidden Single Rule

Killer combination is the most crucial rule. This rule is possibly applied if the cells covered in a cage areallin one row or column and the mathematical operation is addition. There is a unique possible numbers for cells for this killer combination related with the cage size. For example, a clue of target value is 3 in a cage of size 2 can only have 1 or 2 in it. It means possible number from 3 to 9 can be removed from those cells. If a cage with target number is 23 then the only possible number for the cells are 6, 8, or 9. This killer combination for cage size 2 and for any cage value is tabulated in Table 1. The table can be expanded to the other cage size.

TABLE 1.
KILLER COMBINATION FOR CAGE SIZE 2

Cage size	Cage value	Combination
2	3	1/2
2	4	1/3
2	17	8/9
2	16	7/9

X-wing rule stated that when there are only two possible cells for a value in each two different rows and these candidates lie also in the same columns,then all other candidates for this value in the columns can be eliminated. Figure 5 shows an example of this rule.

Figure 5. An example of X-Wing Rule

If A turns out to be a **7** then it rules out a **7** at C as well as B. Because A andCD are 'locked' then D must be a **7**. So a 7 MUST be present at A, D or B, C. If this is the case then any other **7**'s along the edge of our rectangle are redundant. We can remove the **7**'s marked in the green squares.

B. Genetic algorithm

Genetic algorithms (GA) is a kind of generate and test heuristics techniques. It was inspired by the behaviour of natural systems; the terminology used to describe them is a mix from both biological and computer fields. A genetic algorithm manipulates strings of information, usually called chromosomes. These encode potential solutions to a given problem. Chromosomes are evaluated and assigned a score (fitness value) in terms of how well they solve the given problem according to criteria defined by the programmer. These fitness values are used as a probability of survival during a round of reproduction. New chromosomes are produced by combining two (or more) parent chromosomes.

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This process is designed to lead to a succession of fitter offspring, each encoding better solutions, until an acceptably good solution is found.

The working principle of a GA is illustrated below. The major steps involved are the generation of a population of solutions, finding the objective function and fitness function and the application of genetic operators. These aspects are described briefly in the subsection below.

```
Formulate initial population
Randomly initialize population
Repeat
    Evaluate objective function
    Apply genetic operators
        Reproduction
        Crossover
        Mutation
Until stopping criteria
```

III. MODELLING OF KENKEN PUZZLE

Ken-ken puzzle of size n x n has n^2 cells. Cell located in row, b and column, k is labelled as $C_{b,k} = bn + k$ and the value of that cell is $V(C_{b,k}) \in \{1,2,\ldots,n\}$. Ai[th]-cagelabelled A_i is a set of cells, $A_i = \{C_{b,k}\}$. It correlates with one arithmetic operator $o_i \in \{+,-,x,\div\}$ and one target number $H_i \in N$. This is the three rules in making the Ken-ken problem $|A_i| = 1 \rightarrow O_i = \phi$, $O_i \in \{-,\div\} \rightarrow |A_i| = 2$ and $\forall C_{b,k} \rightarrow C_{b,k} \in \exists! A_i$. The purpose of the puzzle is to find the value $V(C_{b,k})$ and meet the constrains :

1. $|A_i| = 1 \wedge C_{b,k} \in A_i \rightarrow V(C_{b,k}) = H_i$
2. $O_i \in \{-,\div\} \wedge A_i = \{C_{a,b}, C_{p,q}\} \rightarrow |V(C_{a,b}) - V(C_{p,q})| = H_i$
3. $O_i \in \{\div\} \wedge A_i = \{C_{a,b}, C_{p,q}\} \rightarrow V(C_{a,b}) / V(C_{p,q}) = H_i$
4. $O_i \in \{+\} \rightarrow \sum_{C_{b,k} \in A_i} V(C_{b,k}) = H_i$
5. $O_i \in \{x\} \rightarrow \prod_{C_{b,k} \in A_i} V(C_{b,k}) = H_i$

The flow chart of solving Ken-ken puzzle is represented in figure 6.

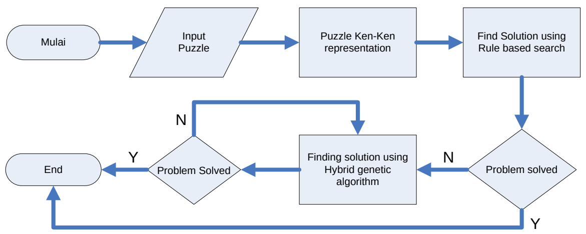

Figure 6. Flow chart of Solving Kenken Puzzle

Rule based search is started by assuming all unknown value of cells by all possible values to fill that cell without breaking the constraints, $P(C_{b,k}) = \{1,2,\ldots,n\}$. after the value of one cell has been determined, the possible values of a certain cells respectively were updated. For example using naked single rule is stated as equation 1, will cause all possible values for all cells in a row and in a columnis updated stated in equation 2 and 3. Applying naked pair is stated in equation 4 for row and 5 for column. Other rules are also defined.

$$|P(C_{b,k})| = 1 \;\wedge\; x \in P(C_{b,k}) \rightarrow V(C_{b,k}) = x \qquad (1)$$

$$(V(C_{b,k}) = x) \wedge (\forall a \in \{1,2,\ldots,n\}) \rightarrow P(C_{a,k}) = P(C_{a,k}) - \{x\} \qquad (2)$$

$$(V(C_{b,k}) = x) \wedge (\forall q \in \{1,2,\ldots,n\}) \rightarrow P(C_{b,q}) = P(C_{b,q}) - \{x\} \qquad (3)$$

$$|P(C_{b,k1})| = |P(C_{b,k2})| = 2 \wedge P(C_{b,k1}) = P(C_{b,k2}) \rightarrow P(C_{b,q}) = P(C_{b,q}) - P(C_{b,k1}) \qquad (4)$$

$$|P(C_{b1,k})| = |P(C_{b2,k})| = 2 \wedge P(C_{b1,k}) = P(C_{b2,k}) \rightarrow P(C_{p,k}) = P(C_{p,k}) - P(C_{b1,k}) \qquad (5)$$

Hybrid genetic algorithm is used while the puzzle still is not able be solved after exercising all the rules repetitively. It started by encoding the chromosome. One chromosome consists of k number of segments, $m \leq n$.One segment is a set of genes that of unsolved cells in that segment.

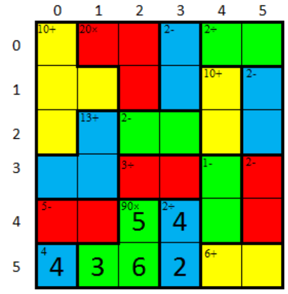

Figure 7. An example of Kenken puzzle 6x6

A Segment represents a row or a column. Segments are ordered in a chromosome. An example of one chromosome from Kenken puzzle in Figure 7 is *34 35 | 28 29 24 25 | 0 4 5 1 2 3 | 11 6 9 7 8 10 | 12 14 15 17 16 13 | 20 18 19 23 21 22.*

Objective function will be calculated after the generation value of gene on chromosome has been done. Value for gene j-th on a chromosome was represented byw_j. w_j isset 0 if gen is not solved, otherwise is set 1. For a chromosome with number of genes k, fitness function is shown by the equation below.

$$x_j = \begin{cases} 0 & , w_j = 0 \\ 1 & , w_j \neq 0 \end{cases} \qquad fitness = \frac{\sum_{j=0}^{k} x_j}{k} \qquad (6)$$

Consequently, the solution of the puzzle is to find out the chromosome with the fitness value is 1.

In reproduction process, crossover, two chromosomes, parents, will be crossed to produce another two chromosomes, children,with N-segments crossover methodology. This process is shown in Figure 8.

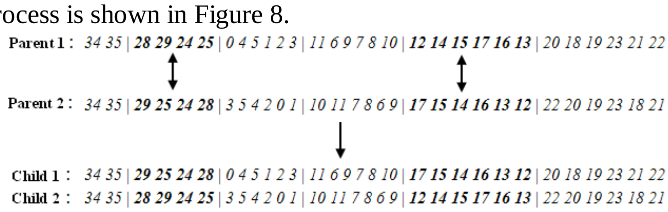

Figure 8. N-segments cross over

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Exchange mutation is used to get another possible chromosome. Mutation is done between genes in the same segment. Figure 9 is an example of a mutation process between two genes on the same segment.

34 35 | 28 ***29*** *24* ***25*** *| 0 4 5 1 2 3 | 11 6 9 7 8 10 | 12 14 15 17 16 13 | 20 18 19 23 21 22*

34 35 | 28 ***25*** *24* ***29*** *| 0 4 5 1 2 3 | 11 6 9 7 8 10 | 12 14 15 17 16 13 | 20 18 19 23 21 22*

Figure 9. An example of mutation process

IV. EXPERIMENT RESULTS

Two types of experiments were conducted. They are to know how fast the system can solve the puzzle accordance to the size and the level of the difficulty of the puzzle. Size variations of Kenken puzzle were experimented 5 × 5, 7 × 7 and 9 × 9with variation of level of difficulty easy, medium, hard and expert. For each category, 10 sample puzzles are used. Some of the puzzle can be solved only by rule based where as the other should be with hybrid genetics algorithm. Table 2 showed the percentage of puzzle that can be solved only by using rule base search.

TABLE 2.
PERCENTAGE OF PROBLEMS THAT CAN BE SOLVED USING RULE BASE SEARCH

Size	Level of difficulty			
	Easy	Medium	Hard	Expert
5 × 5	100%	80%	50%	0%
7 × 7	100%	60%	0%	0%
9 × 9	100%	90%	0%	0%

Average time (in millisecond) of 10 attemptsfor a certain puzzleper unit cell can be seen in Table 3.

TABLE 3.
AVERAGE TIME TO EACH CELL IN MILLISECOND

Size	Num	Level of difficulty			
		Easy	Medium	Hard	Expert
5x5	1	1.4800	0.5600	1.2000	17.4400
	2	0.4800	0.4800	0.5200	33.1600
	3	0.4800	0.4800	0.5600	1.1200
	4	0.4800	0.5200	0.5200	4.1600
	5	0.4800	0.5200	0.5200	1.4800
	6	0.6400	0.4800	0.9600	1.0800
	7	0.4800	1.2000	1.7600	0.9600
	8	0.4800	0.8000	0.7200	1.2400
	9	0.6000	0.4800	2.6800	1.4400
	10	0.4400	0.4800	0.9600	3.0400
7x7	1	0.6939	24.9796	3.1694	1.2429
	2	0.7551	2.3673	1.8143	7.0673
	3	0.6939	0.9796	4.9755	3.0163
	4	0.6735	2.6327	2.4347	1.5918
	5	0.6735	0.7143	2.0041	1.7449
	6	0.7755	0.6939	5.6959	1.0184
	7	0.7347	0.7347	9.3796	2.5531
	8	0.7347	0.7959	119.7704	1.8878
	9	0.7347	0.6735	8.2673	1.3592
	10	0.6735	0.6735	1.0082	96.3367
9x9	1	1.0864	0.9753	31.5099	7.3457
	2	0.8642	0.9877	160.0198	113.6543
	3	0.9753	1.2716	0.7568	0.9457
	4	1.1481	1.8025	23.8963	24.1086
	5	0.8272	0.9259	5.3914	285.7148
	6	0.8765	1.2963	0.8346	0.9840
	7	1.1975	1.3951	0.7049	374.7716
	8	0.8395	1.0000	3.5605	1.6210
	9	0.8889	1.1605	0.9272	28.4617
	10	0.9012	0.9877	1.2630	37.7383

Analyzing the data with F-test, it proved that completion time was independent with puzzle size for any level difficulty. F calculation for level difficulty easy, medium, hard and expert is 2.2, 0.3, 0.3 and 0.8 where as F table is 4.74. It means that for any type of level difficulty, the average time to solve each cell is independence with the size of the puzzle. Further investigation showed that there is no significant different on average elapse time to solve each cell between level difficulty easy and medium however there was between easy and expert. It was caused by most of easy and medium problems could be solved only by using rule based search. On the other hand, all expert problems used hybrid genetic algorithm. Interestingly, there was no significant different elapse time between level difficulty hard and expert.

V. CONCLUSIONS

Through these experiments, hybrid genetic algorithm system can solve kenken puzzle successfully. Kenken puzzle level of difficulty expert can not be done only by using single square, naked pair, naked triple, evil twin, hidden single, killer combination, and X-wing rules. It needs to applies others sophisticated logic rules.

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