 ISSN 2301-6590

The implementation of Secton Method for Solving Systems of Non Linear Equations

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Abstract—*Newton method is a famous method for solving non linear equations numerically. This method is also suitable for solving systems of non linear equations. However, this method has a limitation because it requires the derivative of the non linear function which constructing the systems. The need of derivation some times lead to a problem. Secton method is a combination of Newton method and Secant method. Secton method omit the need of derivation of function. The performance of Secton method is almost same with Newton method. In this paper, Secton method is used to solve a non linear systems of equation. The experiments showed that Secton method could work well for solving non linear systems of equations.*

Keywords-component; Newton method; secant method; non linear equation

2

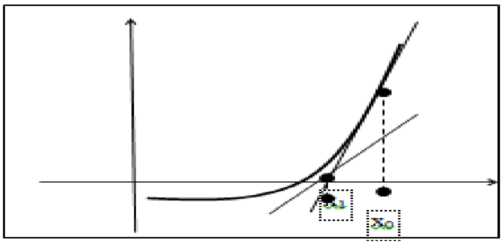

Figure 1. Newton method

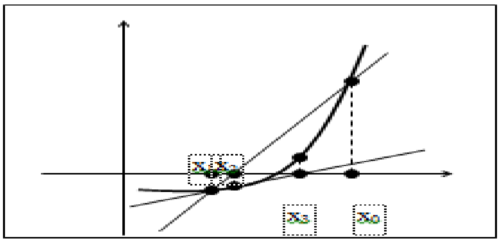

Figure 2. Secant method

ISSN 2301-6590

1 Introduction

The solution of an equation $f(x)=0$ is an intersection point between the graph of $y=f(x)$ and the X axis. Assuming the initial value of the solution is x_0, iteratively the solution could be found by using Eq. 1 as illustrated in Figure 1 which is known as Newton method [1].

$$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}$$

The use of Newton method creates another problem. The derivation of some function is difficult. Secant Method solve this problem by using an approximation of the derivative value ($f(x_n)$, that is the slope of a line connecting two points on the function. Secant method need two initial points. It is shown in Figure 2, by using two initial points x_0 and x_1 the solution could be found using Eq. 2 [1].

$$x_{n+1}=x_n-f(x_n)\frac{x_n-x_{n-1}}{f(x_n)-f(x_{n-1})}$$

2 SECTON METHOD

Numerical solution is found by iterating an initial value which moves to approximate the solution. The iteration process is stopped when the iteration reach the error tolerance (ε), otherwise no solution can be found.

Secton method is developed by using ε in the determination of derivative value of the function to be solved. A derivative of a function $f(x)$ is calculated using [3] :

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

If ε is very small, then derivative value of $f(x)$ at $x=x_n$ is

$$f'(x_n)=\frac{f(x_n+\varepsilon)-f(x_n)}{\varepsilon}$$

Applying (4) into (1) gives Secton method :

$$x_{n+1}=x_n-f(x_n)\frac{\varepsilon}{f(x_n+\varepsilon)-f(x_n)}$$

The performance of Secton method for solving an unstable function, namely Wilkinson polynomial

$$w(x)=\prod_{i=1}^{20}(x-i)$$

has been discussed [4].

The complete solution of the Wilkinson polynomial by using Secton method can be seen in Table I. All the root of the Wilkinson polynomial can be found.

3 NEWTON METHOD FOR SOLVING SYSTEM OF NON LINEAR EQUATIONS

Consider the following system of non linear system

$$f(x,y)=0$$

(7)

$$g(x,y)=0$$

The graph of $z=f(x,y)$ is a surface in xyz-space. The solution of $f(x,y)=0$ lie on the intersection of the *xy*-plane and the graph of $z=f(x,y)$. The intersection is called “zero curve” of $f(x,y)$. The solution of $g(x,y)=0$ lie on the zero curve of $g(x,y)$ [1].

Let (x_0,y_0) be an initial guess for a solution for the system (7). The approximation is constructed by using a tangent plane at $(x_0,y_0,f(x_0,y_0))$, namely $z=p(x,y)$ having

 ISSN 2301-6590

$$p(x,y) \equiv f(x_0,y_0) + (x-x_0)f_x(x_0,y_0) + (y-y_0)g_y(x_0,y_0)$$
(8)

$$f_x(x,y) = \frac{\partial f(x,y)}{\partial x}$$

$$f_y(x,y) = \frac{\partial f(x,y)}{\partial y}$$

The above construction can be applied to the surface $z = g(x,y)$.

$$q(x,y) \equiv g(x_0,y_0) + (x-x_0)g_x(x_0,y_0) + (y-y_0)g_y(y-y_0)$$
(9)

$$g_x(x,y) = \frac{\partial g(x,y)}{\partial x}$$

$$g_y(x,y) = \frac{\partial g(x,y)}{\partial y}$$

The solution of the system is the intersection of the zero curve. Let $(x - x0=\delta x$ and $y-y0=\delta y$.

Solving the following equation

$$f(x_0,y_0) + \delta_x f_x(x_0,y_0) + \delta_y f_y(x_0,y_0) = 0 \qquad (10)$$

$$g(x_0,y_0) + \delta_x g_x(x_0,y_0) + \delta_y g_y(x_0,y_0) = 0$$

will get δ_x and δ_y. The next guess will be [1]$x_1 = x_0 + \delta_x$

$$y_1 = y_0 + \delta_y$$

This kind of solution is a generalization of Newton Method [1]. By using this step the *n+1*th guess can be determined, namely

$$x_{n+1} = x_n + \delta_x$$

$$y_{n+1} = y_n + \delta_y$$

where δ_x and δ_y is obtained from the solution of

$$f(x_n,y_n) + \delta_x f_x(x_n,y_n) + \delta_y f_y(x_n,y_n) = 0 \qquad (11)$$

$$g(x_n,y_n) + \delta_x g_x(x_n,y_n) + \delta_y g_y(x_n,y_n) = 0$$

4 THE IMPLEMENTATION OF SECTON METHOD

Secton method is used to calculate $f_x(x,y)$, $f_y(x,y)$, $g_x(x,y)$ and $g_y(x,y)$, without any process of derivation. Here is the formulae :

$$f_x(x,y) = \frac{f(x+\varepsilon,y) - f(x,y)}{\varepsilon}$$

$$f_y(x,y) = \frac{f(x,y+\varepsilon) - f(x,y)}{\varepsilon}$$

$$g_x(x,y) = \frac{g(x+\varepsilon,y) - g(x,y)}{\varepsilon}$$

$$g_y(x,y) = \frac{g(x,y+\varepsilon) - g(x,y)}{\varepsilon}$$

The general form of the solution can be constructed. Suppose there is a system of *n* nonlinear equations [1].

$$\begin{gathered} F_1(x_1,\ldots,x_n) = 0 \\ \vdots \\ F_n(x_1,\ldots,x_n) = 0 \end{gathered}$$
(12)

Define

$$x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$$

$$F(x) = \begin{bmatrix} F_1(x_1, \dots, x_n) \\ \vdots \\ F_n(x_1, \dots, x_n) \end{bmatrix}$$

$$F'(x) = \begin{bmatrix} \frac{\partial F_1}{\partial x_1} & \cdots & \frac{\partial F_1}{\partial x_n} \\ \vdots & \vdots & \vdots \\ \frac{\partial F_n}{\partial x_1} & \cdots & \frac{\partial F_n}{\partial x_n} \end{bmatrix}$$

By implementing Secton method, $F'(x)$ can be replaced with

$$F'(x) = \left[\frac{\partial F_i}{\partial x_j}\right], \quad i = 1 \dots n, j = 1 \dots n$$

Where

$$\frac{\partial F_i}{\partial x_j} = \frac{F_i(x_1, \dots, x_j + \varepsilon, \dots, x_n)}{\varepsilon}$$

5 TESTING

The implementation of Secton method for solving system of non linear equations is tested to solve

$$x^2 + 4y^2 - 9 = 0$$

$$18y - 14x^2 + 45 = 0$$

The solutions of this system are $(1.203167, -1.374081)$, $(-1.203167, -1.374081)$, $2.1372167, 1.052652)$ and $(-2.1372167, 1.052652)$.

Table II shows the iteration of (10,10) as an initial guess. Various initial guesses and its corresponding solutions are showed in Table III.

6 CONCLUSION

The implementation of Secant method can a find solution of system of nonlinear equations without any process of derivation of the functions composing the system..

TABLE I THE COMPLETE SOLUTION OF WILKINSON POLYNOMIAL BY USING SECTON METHOD

X0	Solution	iteration	X0	solution	iteration	X0	solution	iteration
-1.25	1.0000	12	6.75	7.0000	3	14.75	15.0000	5
-0.75	1.0000	11	7.25	7.0000	4	15.25	15.0000	4
-0.25	1.0000	9	7.75	8.0000	4	15.75	16.0000	5
0.25	1.0000	8	8.25	8.0000	4	16.25	16.0000	4
0.75	1.0000	5	8.75	9.0000	4	16.75	17.0000	5
1.25	20.0000	16	9.25	9.0000	3	17.25	17.0000	4
1.75	2.0000	5	9.75	10.0000	4	17.75	18.0000	5
2.25	1.0000	4	10.25	10.0000	4	18.25	18.0000	4
2.75	3.0000	4	10.75	11.0000	4	18.75	20.0000	5
3.25	3.0000	5	11.25	11.0000	4	19.25	19.0000	4
3.75	4.0000	4	11.75	12.0000	3	19.75	1.0000	18
4.25	4.0000	5	12.25	12.0000	4	20.25	20.0000	5
4.75	5.0000	4	12.75	13.0000	4	20.75	20.0000	8
5.25	5.0000	5	13.25	13.0000	4	21.25	20.0000	8
5.75	6.0000	3	13.75	14.0000	4	21.75	20.0000	11
6.25	6.0000	5	14.25	14.0000	3	22.25	20.0000	12

THE COMPLETE SOLUTION OF WILKINSON POLYNOMIAL BY USING SECTON METHOD

ISSN 2301-6590

(E=0.0001)

	initial points		solutions	
No	x	y	x	y
1	10	10	2.137217	1.052652
2	-10	10	-2.137217	1.052652
3	10	-10	1.203167	-1.374081
4	-10	-10	-1.203167	-1.374081

TABLE I. ITERATION OF AN INITIAL POINT BY USING SECTON METHOD ON A SYSTEM OF NONLINEAR EQUATION (E=0.0001)

iteration	x	y
0	1	-1
1	1.1467	-1.4678
2	1.1882	-1.3994
3	1.2015	-1.3763
4	1.2030	-1.3743
5	1.2032	-1.3741
6	1.2032	-1.3741
7	1.2032	-1.3741
8	1.2032	-1.3741
9	1.2032	-1.3741
10	1.2032	-1.3741

TABLE II. VARIOUS INITIAL POINTS AND ITS SOLUTIONS OF SYSTEM OF NON LINEAR EQUATIONS (E=1X 10^{-15})

7 REFERRENCES

[7] Atkinson, K., 1985, “Elementary Numerical Analysis”, John Wiley &Sons, Inc., New York.

[8] Buchanan, J.L. and Turner, P.R., 1992, “Numerical Methods and Analysis“, McGraw-Hill, New York.

[9] Loomis, L.H., 1982, “Calculus”, 3rd Edition, Addison-Wesley Publishing Company, Sydney.

[10] Rokhman, N., 2011, “Secton : A Combination of Newton Method and Secant Method for Solving Non Linear Equation”, SNATI 2011, UII, Yogyakarta