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Comparative Analysis for The Multi Period Degree Minimum Spanning Tree Problem

Wamiliana, Amanto, and Mustofa Usman
Department of Mathematics, Faculty of Mathematics and Natural Sciences, Lampung University

Abstract-*The Multi Period Degree Constrained Minimum Spanning Tree Problem (MPDCMST) concerns of finding the total minimum cost of networks installation, where the installation process is divided into some periods. In the beginning of installation process, the center of the networks already set (as server, reservoir, etc). The installation process is divided into some period due to some factors, usually fund limitation. During the installation process, the networks is supposed to be maintained its reliability by restrict the numbers of links that can be connected to the node that already in the networks. In this paper we will discuss and improve the performance of WADR1 and WADR2 algorithms by setting the number of elements in the set of vertices that must be in installed in a certain period as a fix number and adding the length of the path in DFS. The result shows that the modifications works better.*

Keywords: *Multi period, degree constrained, minimum spanning tree, comparative analysis*

1. Introduction

Combinatorial optimization problems arise in various applications including communication network design, VLSI design, airline crew scheduling, database query design, transportation network design, etc. In addition, combinatorial optimization problems occur in many diverse areas such as graph theory, linear and integer programming, number theory and artificial intelligence.

A network is a system which involves the movement or flow of some commodity such as products, information, electrical current, mail, people, cars, trains, water, heat, and so on. By using the connections available in the network, the commodity usually originates from the origin (source) and moves to the terminal (sink). Therefore any structure that appears in the form of a system of lines and a system of components having a common purpose is considered a network. For example, a transportation network is a collection of stations or depots that are linked by the railways or roads to enable people or goods to be transported from one station to another. Indeed, telecommunication networks, electrical networks and computer networks are included in such networks [9]

Network design as one of the areas of combinatorial optimization, plays an important role in many real-life applications. In this modern age where accurate models and efficient solution techniques are required, it provides the representation of problems at hand. Some examples of network design include: transportation networks for the movement of commodities; communication networks for the transmission of information; powerful multiprocessor systems for solving complex problems such as radar signal processing and many more [5].

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Multi Period Degree Constrained Minimum Spanning Tree Problem (MPDCMST) is one of network problems that frequently occur in real life situation, especially in networks installation. For example the installation of power supply networks system, the installation of telecommunication networks, computer networks, water supply networks, and so on. However, in reality the installation process needs to be done in some periods, mostly because of the fund limitation. When the installation process only needs one period, the problem is called as Degree Constrained Minimum Spanning Tree Problem [12].

This paper is organized as follow: in Section 1 we give the Introduction about MPDCMST, in Section 2 we give the survey about the methods that already investigated for solving the MPDCMST, in Section 3 we discuss about the algorithm developed, in Section 4 we give the implementations and results, followed by conclusion.

2. The Multi Period Degree Constrained Minimum Spanning Tree Problem

On of the classical problem in networks design is finding The Minimum Spanning Tree (MST) problem. To find a minimum-spanning tree, there are two well-known algorithms: Kruskal's and Prim's However, the earliest algorithm for finding a minimum spanning tree according to Graham and Hell [4] was suggested by Boruvka in 1926 who developed an algorithm for finding the most economical layout for a power-line network [16]

Adding the degree restriction during the construction of the MST will reduced the problem to the Degree Constrained Minimum Spanning Tree (DCMST) problem. The DCMST problem can be formulated as a Mixed Integer Linear Programming as follow [9]:

$$Minimise \quad \sum_{i}^{n}\sum_{j}^{n} c_{ij} x_{ij} \qquad (2.1)$$

subject to

$$\sum_{i,j} x_{ij} = n-1 \qquad (2.2)$$

$$\sum_{i,j \in V'} x_{ij} \le |V'|-1, \qquad \forall \emptyset \ne V' \subseteq V \qquad (2.3)$$

$$1 \le \sum_{j=1, i\ne j} x_{ij} \le b_i \qquad i = 1,2,\ldots,n \qquad (2.4)$$

$$x_{ij} = 0 \text{ or } 1, \qquad 1 \le i \ne j \le n. \qquad (2.5)$$

c_{ij} is the weight (or distance or cost) of the edge (i,j), b_i is the degree bound on vertex i and n is the number of vertices. Constraint (2.2) ensures that (n-1) edges are selected. Constraint (2.3) is the usual subtour elimination constraints. Constraint (2.4) specifies the degree restriction on the vertices. The last constraint (2.5), is just the variable constraint, which restricts the variables to the value of 0 or 1. x_{ij} is 1 if the edge x_{ij} is selected or included in the tree T and 0, otherwise. This formulation is the most common formulation for the DCMST problem.

Some methods already investigated for solving the DCMST, for example: greedy heurietics based on Prim's or Kruskal's algorithms [8], Genetic Algorithm [16] Simulated Annealing [6], Iterative Refinement [1], [2]; Modified Penalty [10],[11],[14]; and Tabu Search [2], [11], [12].

Adding the periods to the DCMST will reduced the problem to The MPDCMST

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Problem. The MPDCMST problem is a problem of determining how many vertices should be installed in a certain period in such a way so that the total cost of installation is minimum. After all periods completed, all vertices in the networks should be connected, and the total cost is the minimum.

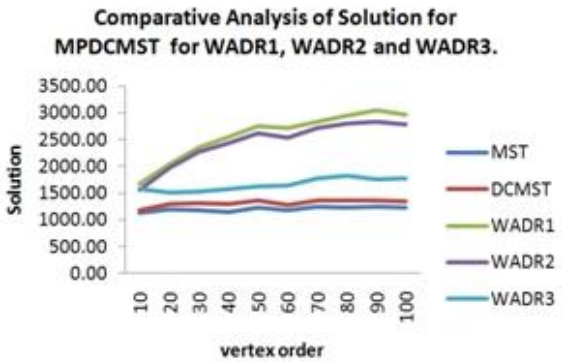
The first method investigated for MPDCMST was proposed by [11] where branch exchange technique used as a hybrid to lagrangean Relaxation, and the method was implemented using vertices varying from 40 to 100; 10 year planning horizon; the time period for activating each terminal is uniformly distributed from 1 to 6; and set vertex 1 as central vertex.

The other type of MPDCMST ws investigated in [13] where in this paper they used one year planning horizon and divided the installation into three periods (for month each) and four periods (three month each). That modification of MPDCMST was made to mimic the real situation in Indonesia where the funding for every project usually divided into three terms or periods. In the method developed they got feasible solution for all data tested. In [5] the method was improved from the method proposed in [13] and tested using some problems taken from TSPLIB. WADR1 and WADR2 algorithm were investigated in [15]. These algorithms adopt and modified Kruskal's algorithms altogether with DFS technique for k = 2, k is the depth of child vertex. In the algorithm they introduced a set HVT_i as a set of vertices that must be already in the networks after period i finished. The use of HVT_i to tackle the problem that some facility (for example hospital, police station, or other public need facilities) must be in the network earlier to handle public needs. The difference between WADR1 and WADR2 lied on the process of installation HVT_1.

In this paper we propose the improve of WADR1 and WADR2 algorithms by setting $HVT_i = 3$, $MaxVT = \left\lfloor \frac{n-1}{3} \right\rfloor$ and set $k \leq 3$.

3. Implementations and Results.

We use complete graph K_n with vertex order n to represent the problem. The data generated assigned for edge weight are uniformly distributed with the weight vary between 1-1000, for every order of the graph we generate 30. That data also used by Junaidi et al (2008). We implement our problem using C++ programming language running on dual core computer with 1.83GHz, 2 GB RAM. The following chart shows the results.



From the chart we can see that the solution of WDAR3 is better than WADR1 and WADR2 and its solutions closer to the lower bound (DCMST).

References

[1]. Boldon, B., N. Deo and Nishit Kumar (1996). ' Minimum Weight degree-constrained spanning tree problem: Heuristics and Implementation on an SIMD parallel machine', *Parallel Computing* **vol. 22**, pp.369 –382.

[2].Caccetta L., and Wamiliana, 2001. Heuristics Algorithms for The Degree Constrained Minimum Spanning Tree Problem, in Proceeding of The International Congress on Modeling and Simulation (MODSIM 2001), Canberra. Editor : F. Ghassemi et al. , pp. 2161-2166.

[3].Deo,N. and Nishit Kumar, 1997. 'Computation of Constrained Spanning Trees: A Unified Approach', *Network Optimization* (Lecture Notes in Economics and Mathematical Systems, Editor : Panos M. Pardalos, et al , Springer-Verlag, Berlin, Germany, pp. 194 – 220.

[4].Graham, R.L., and Hell, P., ' On the history of the Minimum Spanning Tree Problem', 1982. *Mimeographed*, Bell Laboratories, Murray Hill, New Jersey

[5]. Junaidi, A.., Wamiliana, Dwi Sakethi, and Edy Tri Baskoro, 2008. Computational Aspect of Greedy Algorithm for The Multi Period Degree Constrained Minimum Spanning Tree Problem, *Jurnal Sains MIPA*, Special Edition Vol. 14 No. 1. Pp 1-6

[6] Krishnamoorthy, M., A.T. Ernst and Yazid M Sharaila (2001), 'Comparison of Algorithms for the Degree Constrained Minimum Spanning Tree', *Journal of Heuristics*, **Vol. 7**, no. 6, pp. 587-611.

[7].Kawatra, 2002.A multi period degree constrained Minimum Spanning Tree Problem, *European Journal of Operational Research*, Vol **143**, pp. 53 – 63.

[8]. Narula,S. C., and Cesar A.Ho, 1980. ' Degree-Constrained Minimum Spanning Tree', *Computer and Operation Research* , **Vol. 7**,pp. 239-249

[9]. Wamiliana,2002. Combinatorial Methods for Degree Constrained Minimum Spanning Tree Problem, Doctoral Thesis, Department of Mathematics and Statistics, Curtin University and Technology, Australia.

[10]. Wamiliana, 2002. 'The Modified Penalty

Methods for The Degree Constrained Minimum

Spanning Tree Problem', *Jurnal Sains dan Teknologi*, **Vol. 8**, pp.1-12.

[11]. Wamiliana, 2004. 'Solving the Degree Constrained Minimum Spanning Tree Using Tabu and Penalty Method', *Jurnal Teknik Industri*, pp.1-9.

[12] Wamiliana and Caccetta. 2003. "Tabu search Based Heuristics for the Degree Constrained Minimum Spanning Tree Problem,Proceeding of South East Asia Mathematical Society, pp. 133-140

[13] Wamiliana, Dwi Sakethi, Akmal J, and Edy Tri Baskoro, 2005. The Design of Greedy Algorithm for Solving The Multi Period degree Constrained Minimum Spanning Tree Problem, *Jurnal sains dan Teknologi* **Vol 11** No. 2, pp. 93 – 96.

[14] Wamiliana and L. Caccetta, 2006. Computational Aspects of The Modified Penalty for Solving The Degree Constrained Minimum Spanning Tree

ISSN 2301-6590

Problem, *Journal of Quantitatve Methods* **Vol. 2** No. 2 pp. 10 – 16.

[15] Wamiliana, Dwi sakethi, and Restu Yuniarti, 2010. Computational Aspect of WADR and WADR2 Algorithms for The Multi Period Degree Constrained Minimum Spanning Tree Problem, Proceeding SNMAP 2010, pp. 208 – 214.

[16] Wamiliana, 2013. Computational Aspect of Modified Kruskal Algorithms for Degree Restricted Minimum Spanning Tree Problem, The 5th International Conference on Numerical Optimizations and Operations Research (ICNOOR-V), Banda Aceh, 26 – 28 June, 2013.

[17] Zhou G., and Mitsuo Gen, 1997. A Note on Genetic Algorithms for The Degree Constrained Minimum Spanning Tree Problem, *Networks,* Vol. 30, pp. 91-95.